

The graph of  $f(x)$  is shown on the right.

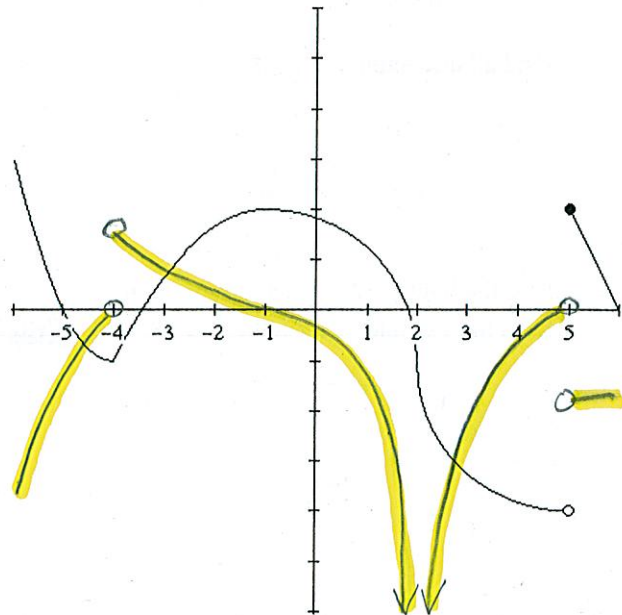
SCORE: \_\_\_ / 20 POINTS

- [a] List all  $x$ -values for which  $f'(x)$  does not exist.  
For each  $x$ -value, give a **brief** reason why not.

$$x = -4 \text{ (CUSP)}$$

$$x = 2 \text{ (VERTICAL T.L.)}$$

$$x = 5 \text{ (DISCONTINUITY)}$$



- [b] Sketch a graph of  $f'(x)$  on the same axes.

Pat is taking vitamin pills regularly. The time between pills depends on the amount of vitamin in each pill.

SCORE: \_\_\_\_ / 15 PTS

Let  $t = g(v)$ , where  $t$  is the time between pills (in hours), and  $v$  is the amount of vitamin per pill (in grams).

[a] What are the units of  $g'(v)$  ?

HOURS/GRAM

[b] What does  $g'(6) = 8$  mean ? Give the units for all numbers in your answer.

IF EACH PILL HAS 6 GRAMS OF VITAMIN,  
THEN FOR EACH ADDITIONAL GRAM PER PILL,  
THE TIME BETWEEN PILLS SHOULD INCREASE 8 HOURS

[c] Is there a value of  $v_0$  for which you would expect  $g'(v_0) < 0$  ? Why or why not ?

NO. IF EACH PILL HAS MORE VITAMINS,  
THE TIME BETWEEN PILLS SHOULD INCREASE,  
NOT DECREASE

Let  $f(x) = \frac{2-x}{x^3-3x^2}$ .

SCORE: \_\_\_\_ / 20 PTS

- [a] Find all discontinuities of  $f$ .

$$x^3 - 3x^2 = 0$$

$$x^2(x-3) = 0$$

$$x = 0, 3$$

- [b] Find the limit of  $f$  at each discontinuity.

Each limit should be a number,  $\infty$  or  $-\infty$ . Write DNE only if the other possibilities do not apply.

$$\lim_{x \rightarrow 0^+} \frac{2-x}{x^2(x-3)} = -\infty$$

$$\frac{2}{0^+(-3)}$$

$$\lim_{x \rightarrow 0^-} \frac{2-x}{x^2(x-3)} = -\infty$$

$$\frac{2}{0^-(-3)}$$

$$\lim_{x \rightarrow 0} \frac{2-x}{x^2(x-3)} = -\infty$$

$$\lim_{x \rightarrow 3^+} \frac{2-x}{x^2(x-3)} = -\infty$$

$$\frac{-1}{9(0^+)}$$

$$\lim_{x \rightarrow 3^-} \frac{2-x}{x^2(x-3)} = \infty$$

$$\frac{-1}{9(0^-)}$$

$$\lim_{x \rightarrow 3} \frac{2-x}{x^2(x-3)} \text{ DNE}$$

State the formal definition of "horizontal asymptote".

SCORE: \_\_\_\_ / 5 PTS

$f$  HAS A HORIZONTAL ASYMPTOTE AT  $y=b$  IFF

$$\lim_{x \rightarrow \infty} f(x) = b \quad \text{OR} \quad \lim_{x \rightarrow -\infty} f(x) = b$$

Find the equation(s) of all horizontal asymptote(s) of  $f(x) = \frac{2-e^{3x}}{7e^{3x}+5}$ .

SCORE: \_\_\_\_ / 15 PTS

$$\lim_{x \rightarrow \infty} \frac{2-e^{3x}}{7e^{3x}+5} = \lim_{x \rightarrow \infty} \frac{\frac{2}{e^{3x}} - 1}{7 + \frac{5}{e^{3x}}} = \frac{0-1}{7+0} = -\frac{1}{7}$$

$$\lim_{x \rightarrow -\infty} \frac{2-e^{3x}}{7e^{3x}+5} = \frac{2-0}{0+5} = \frac{2}{5}$$

$$y = -\frac{1}{7} \quad \text{AND} \quad y = \frac{2}{5}$$

Let  $f(x) = 2x^3 - x^2 - 4$ .

SCORE: \_\_\_\_ / 30 PTS

- [a] Find  $f'(x)$ . No credit will be given for using differentiation shortcuts from chapter 3.

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{2(x+h)^3 - (x+h)^2 - 4 - (2x^3 - x^2 - 4)}{h} \\ &= \lim_{h \rightarrow 0} \frac{2(x^3 + 3x^2h + 3xh^2 + h^3) - (x^2 + 2xh + h^2) - 4 - 2x^3 + x^2 + 4}{h} \\ &= \lim_{h \rightarrow 0} \frac{6x^2h + 6xh^2 + 2h^3 - 2xh - h^2}{h} \\ &= \lim_{h \rightarrow 0} (6x^2 + 6xh + 2h^2 - 2x - h) \\ &= 6x^2 - 2x \end{aligned}$$

- [b] Find the equation of the tangent to the graph of  $f$  at the point where  $x = -1$ .

$$f(-1) = -2 - 1 - 4 = -7 \quad f'(-1) = 6 + 2 = 8$$

$$y + 7 = 8(x + 1)$$

State the Intermediate Value Theorem.

SCORE: \_\_\_\_ / 10 PTS

IF  $f$  IS CONTINUOUS ON  $[a, b]$   
AND  $N$  IS BETWEEN  $f(a)$  AND  $f(b)$   
THEN THERE IS A VALUE  $c \in (a, b)$   
SUCH THAT  $f(c) = N$

Can the Intermediate Value Theorem be used to prove that the equation  $\sec x = -1$  has a solution in the interval  $(\frac{\pi}{3}, \frac{4\pi}{3})$  ? If so, write a proof. If not, explain why not.

SCORE: \_\_\_\_ / 10 PTS

NO,  $\sec x = \frac{1}{\cos x}$  IS NOT CONTINUOUS ON  $[\frac{\pi}{3}, \frac{4\pi}{3}]$   
(AT  $\frac{\pi}{2}$ )



At time  $t$  hours, the position of an object moving in a straight line is  $p(t) = \frac{26+3t}{2+t}$  meters.

SCORE: \_\_\_\_ / 25 PTS

- [a] What is the average velocity of the object from time  $t = 3$  to time  $t = 8$ ? Give the units of your answer.

$$\frac{p(8) - p(3)}{8 - 3} = \frac{\frac{50}{10} - \frac{35}{5}}{5} = \frac{5 - 7}{5} = -\frac{2}{5} \text{ m/hr}$$

- [b] What is the instantaneous velocity of the object at time  $t = 3$ ? Give the units of your answer.

$$\begin{aligned} & \lim_{b \rightarrow 3} \frac{\frac{26+3b}{2+b} - 7}{b-3} \\ &= \lim_{b \rightarrow 3} \frac{26+3b-7(2+b)}{(b-3)(2+b)} \\ &= \lim_{b \rightarrow 3} \frac{12-4b}{(b-3)(2+b)} \\ &= -\frac{4}{5} \text{ m/hr} \end{aligned}$$

$$\begin{aligned} \text{OR } & \lim_{h \rightarrow 0} \frac{\frac{26+3(3+h)}{2+(3+h)} - 7}{h} \\ &= \lim_{h \rightarrow 0} \frac{35+3h-7(5+h)}{h(5+h)} \\ &= \lim_{h \rightarrow 0} \frac{-4h}{h(5+h)} \\ &= -\frac{4}{5} \text{ m/hr} \end{aligned}$$